Axial-Symmetric Boundary Value Problem with Nonlinear Elasticity

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THE application of solutions of linear elastic boundary value problems to the interpretation of the behavior of a number of real materials is a matter of expediency. For a number of materials, for instance, solid propellants, the assumption of a linear elastic stress strain relation is an approximation; even within the range of small strains, it is of interest to determine how the deviation from nonlinearity in a real material affects the stress distribution in a boundary value problem.

The formulation of any nonlinear response must be in invariant terms.[1] For a strain-softening material that has a linear volumetric response, a simple form of nonlinearity is obtained by assuming the shear modulus G to be a function of the second deviatoric strain invariant I_2', $G = G_0(1 - CI_2')$, where G_0 and C are constants that reflect the shear modulus as the magnitude of the strain approaches zero and the amount of the nonlinearity, respectively, and $I_2' = \frac{1}{2}e_{ij}\,e_{ij}$.

The authors[2] have investigated techniques for the solution of a nonlinear thick-walled cylinder (as just described) subjected to internal pressure and contained by a thin elastic shell with Young's modulus E under conditions of plane strain. Figure 1 shows some of the results of this investigation which do not appear in Ref. 2. These results were obtained by writing the equilibrium equation in terms of the displacement and subsequently applying finite difference techniques. An iterative procedure was programmed for a digital computer in which at each step the equation was linearized by determining the elastic constants from the previous step. For the largest pressure 60 iterations were used, which required about two hours on an IBM 1620 when the cylinder was divided into 40 sections.

Figure 2 shows the relationship between the second deviation stress invariant, $J_2' = \frac{1}{2}\,s_{ij}\,s_{ij}$, and I_2'. This curve is very nearly linear for small values of I_2' for which the behavior of the cylinder differs only slightly from the behavior of a linear cylinder. It is seen (Fig. 1) that, for increasing pressure, the nondimensional circumferential stress near the inside surface decreases in a manner quite similar to the elastic-plastic cylinder. However, for larger pressures the stress becomes negative and there is a significant corresponding increase in longitudinal stress (not shown). For increasing pressure, the material approaches a state of hydrostatic stress.

In the example shown $a/b = 0.5$, $t/b = \frac{1}{40}$, $E/G_0 = 100$, $K_0/G_0 = 1$, and $C = 2 \times 10^6$.

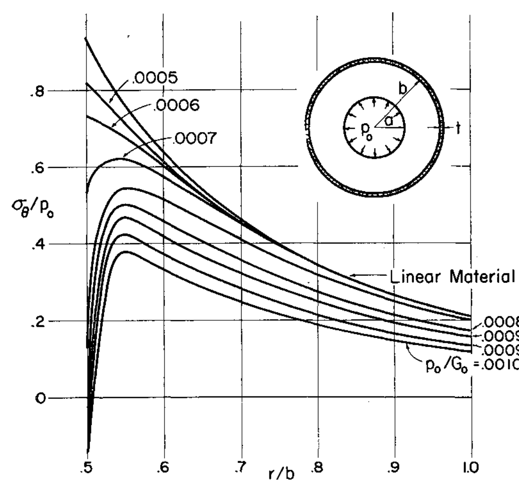

Fig. 1 Circumferential stress diagrams

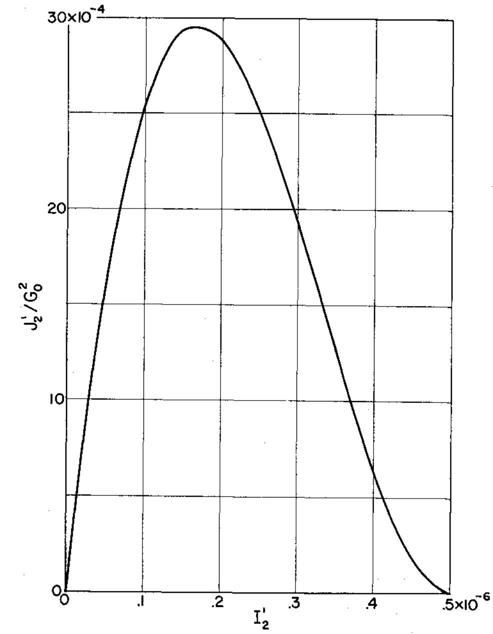

Fig. 2 Deviatoric stress invariant vs deviatoric strain invariant

References

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Electromagnetic Probe for the Measurement of Hypersonic Flow Velocity at a Point

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IN the past it has not been possible to measure freestream velocity at a point in a hypersonic flow field without undue difficulty. At the Naval Ordnance Laboratory, a feasibility study was made to determine whether such a velocity-measuring technique could be developed. A method was devised to make use of a rapidly varying high-intensity magnetic field in the stagnation region about a probe to produce a disturbance in the standing shock. This disturbance is observable by high-speed schlieren photography as it travels outward along the standing shock.

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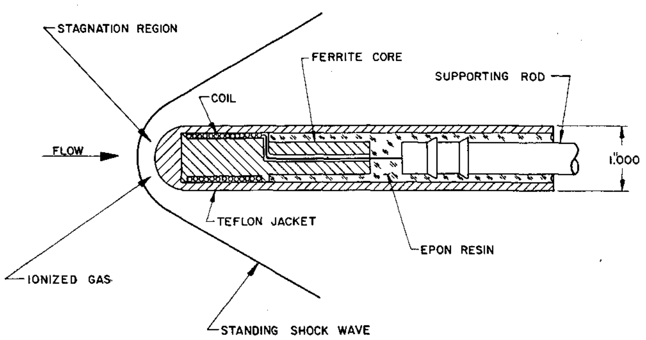


Fig. 1 Cross section of magnetic probe in steady hypersonic flow

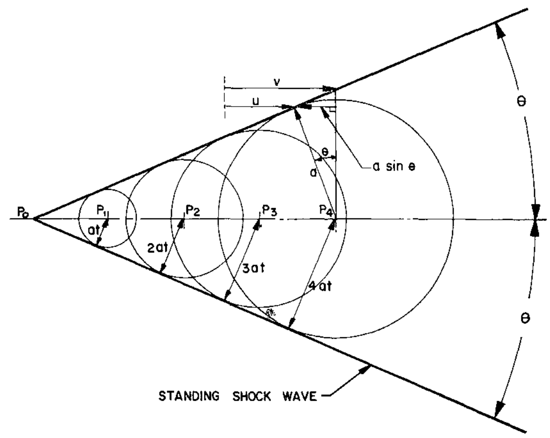


Fig. 2 A point disturbance in hypersonic flow

The probe is a cylinder, 1 in. in diameter, with a hemispherical end. This probe consists of a copper coil of 50 turns wound about a ferrite core and contained within a thin teflon shell (see Fig. 1).

When the probe is positioned in steady hypersonic flow, it effects a standing bow shock wave that is a smooth surface of revolution (see Fig. 1). This standing shock can be represented as the envelope of a series of compression waves that expand at the velocity of sound a and the centers of which are transported back with the freestream flow velocity V.[1] As an example, consider a point disturbance in hypersonic flow. The resulting spherical-compression wave front travels away from the disturbing center with the velocity of sound. Meanwhile, the entire expanding wave front travels along with the hypersonic freestream flow velocity. The net result is an effective "standing wave" about the disturbance which is the envelope of these expanding spheres (see Fig. 2). The velocity with which a point on the standing shock travels is equal to the vector sum of freestream flow velocity and the velocity of expansion from the displaced center, which is the sound speed of the undisturbed flow. The net velocity U with which the point moves back along the flow direction is given by

$$U = V - a \sin\theta \tag{1}$$

where θ is the shock angle at the observed point. Thus, to find the freestream velocity at a point, it is necessary only to measure the net axial velocity and the shock angle, the latter contributing only a small corrective term at high Mach numbers.

When a large axial magnetic field is applied to the end of the probe, any ionized gas particles present in the stagnation region having a velocity component normal to the axis of the probe (as do the particles escaping the region) experience a Lorentz force. The effect of this force is to impede the escape of the ionized particles from the stagnation region. Therefore, there is an increase in the stagnation pressure and temperature which is attended by a subsequent increase in the detachment distance of the shock wave.

If the magnetic field is a very rapidly varying pulse of high intensity (produced in this experiment by the discharge of 10,000 v from a 0.1-μf capacitor through the probe coil in 1.75 μsec to yield a peak magnetic field strength of 10 kgauss), there is a sudden disturbance in the shock detachment distance. The disturbance is propagated along the shock as a pulse or spike. This pulse, then, distinctly marks a particular position of the standing shock as it travels outward with an effective velocity component at each particular point along the probe axis as given by Eq. (1). The motion of this disturbance along the shock is observable by means of a high-speed framing camera in a schlieren optical system.

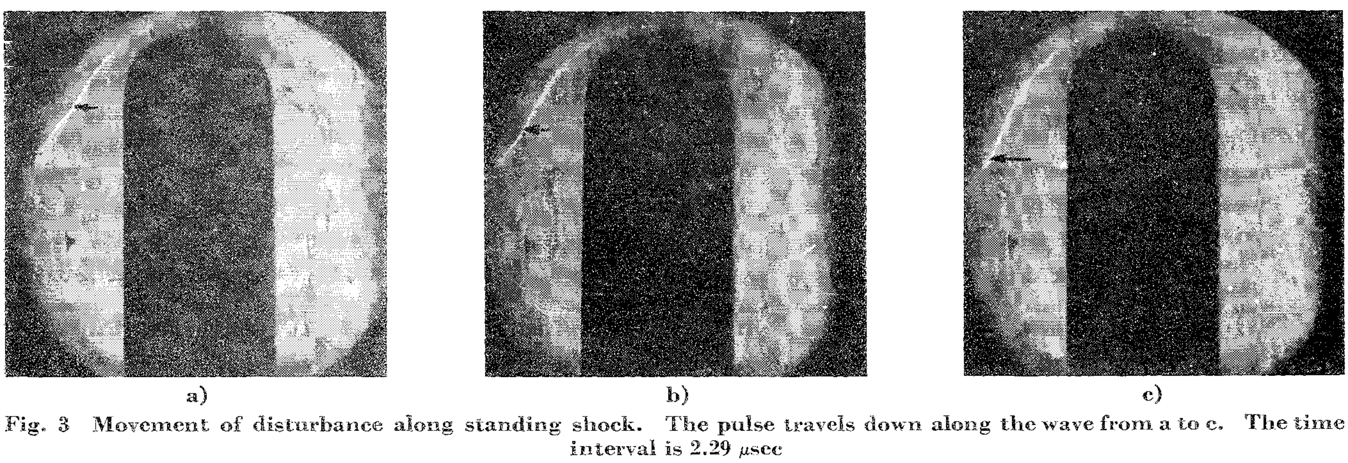
The experiments were performed in the 20-mm Shock Tunnel Facility[2] at the Naval Ordnance Laboratory. The disturbance definitely was observed as it propagated along the standing shock. Data corresponding to a typical experimental run are given in Fig. 3, where the velocity of the disturbance was measured to be 9370 fps corresponding to a freestream velocity of 9940 fps.‡

The technique of using an electromagnetically produced shock wave disturbance to measure not only freestream flow velocity, but also other properties of a hypersonic fluid, suggests further investigation. Little has been done, for example, with either the amplitude and attenuation of the pulse or the suitability of other aerodynamic shapes for the production of such disturbances.

References

[1] Carafoli, E., *High Speed Aerodynamics* (Pergamon Press Inc., New York, 1956), pp. 89–90.

‡ Because of a divergent flow field, the pulse was observed to accelerate as it traveled outward.



a) b) c)

Fig. 3 Movement of disturbance along standing shock. The pulse travels down along the wave from a to c. The time interval is 2.29 μsec

[2] Aronson, P. M., Marshall, T., Seigel, A. E., Slawsky, Z. I., and Smiley, E. F., "Shock tube wind tunnel research at the U. S. Naval Ordnance Laboratory," *Proceedings of the Second Shock Tube Symposium*, Air Force Special Weapons Center Rept. SWR-TM-58-3 (1958), pp. 4–27.

Conical Flarings in Uniform Supersonic Flow at Zero Angle of Attack

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Numerical calculations were carried out on the Illiac,† based on the method of characteristics for axially symmetric supersonic flow and the oblique-shock relations, disregarding the effects of vorticity behind the (slightly) curved shock fronts. In Ref. 1 surface-pressure coefficients, drag coefficients, and shock-front configurations were presented in graphical form as functions of a dimensionless length coordinate for half-cone angles of 5°, 10°, 15°, and 20° and approaching Mach numbers of 1.5, 2.0, 2.5, 3.0, and 3.5 (see Fig. 1 for geometry and nomenclature).

Hypersonic-similarity concepts[2] can be used to achieve a more generalized interpretation of these data and at the same time to illustrate the accuracy in their use for the whole range of cone angles and Mach numbers covered by the original computations.

Figure 2 provides information on the surface-pressure coefficient

$$C_{ps} = (p_s - p_\infty)/\tfrac{1}{2}\rho_\infty V_\infty^2 = C_{ps}(R/R_0, M_\infty, \theta_s)$$

by representing, in graphical form, $C_{ps}/\sin\theta_s^2$ as a function of the modified hypersonic-similarity parameter $K = (M_\infty^2 - 1)^{1/2}\tan\theta_s$ for various values of R/R_0.

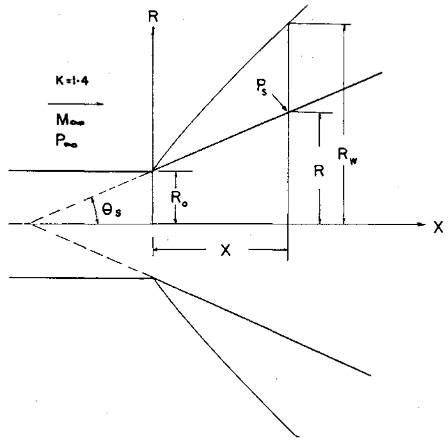

Fig. 1 Conical flaring in supersonic flow

Figure 3 yields information in a similar fashion on the drag coefficient, established on the basis of the maximum cross-sectional area of the flaring,

$$C_{DR} = \frac{1}{(R/R_0)^2}\int_1^{R/R_0} C_{ps} d\left(\frac{R}{R_0}\right)^2$$

The shock-front geometry also can be presented conveniently in a form suggested by supersonic-hypersonic similarity concepts, as shown in Fig. 4, where R_W/R_0 is plotted against $x/R_0 \tan\theta_s$ for values of $0.1 < K < 0.6$.

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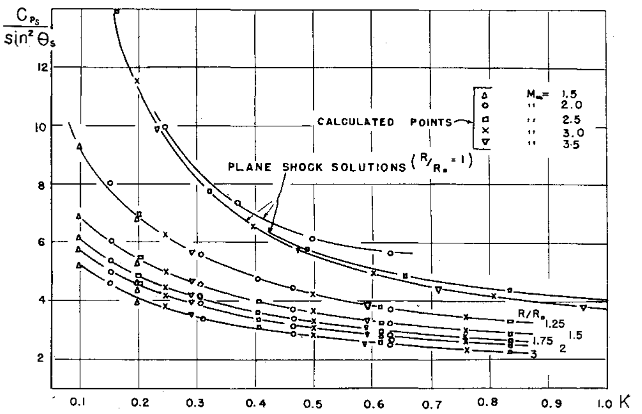

Fig. 2 Pressure coefficient for conical flarings in supersonic-hypersonic similarity presentation

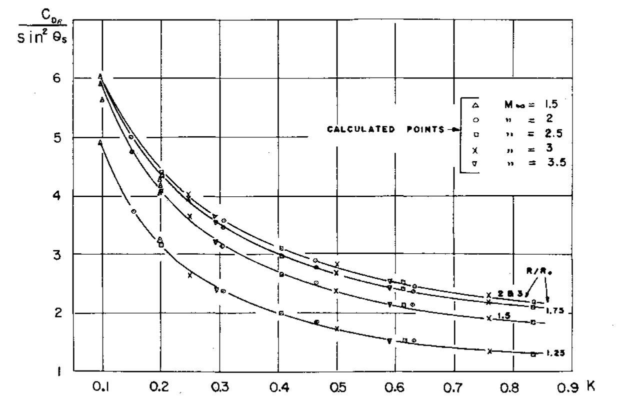

Fig. 3 Drag coefficient for conical flarings in supersonic-hypersonic similarity presentation

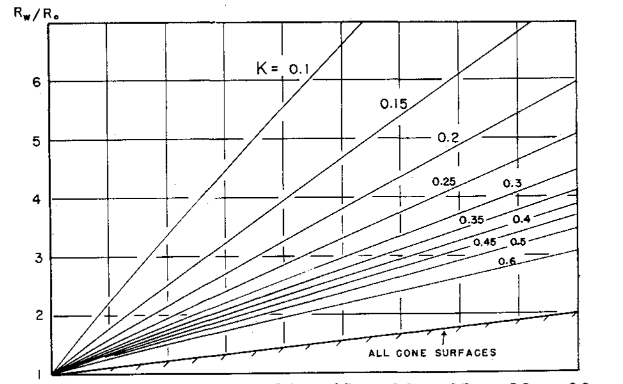

Fig. 4 Shock-front configuration for conical flarings in supersonic flow at various similarity parameter values

Correlation obtained by presenting the original data of Ref. 1 in terms of the similarity concepts is indicated by showing the individually calculated points in addition to the faired curves. The latter are based predominantly on those cases in which the small-disturbance aspects of the similarity concepts were satisfied best.

The asymptotic tendency of drag coefficients toward that for cones obtained by Newton's impact theory is well born out in Fig. 3, where, for large values of K and R/R_0, the coefficient $C_{DR}/\sin^2\theta_s$ approaches the value 2.

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[2] Van Dyke, M. D., "Applications of hypersonic small-disturbance theory," J. Aeronaut. Sci. 21, 179 (1954).